FUNCTIONAL READINGS III

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1 *Today's roadmap*

- Projecting partiality in functional readings.
- Partiality, partitioning, and presupposition projection in questions.
- Question composition and the copy theory of movement.
- Functional readings via complex copies.
- Weak crossover.
- Comparison with Engdahl.
- Functional readings without covert pronouns.

2 *Projecting partiality*

> DESIDERATUM: answers should vary across potentially *partial* functions f, which map girls to pictures of themselves.

(1) a. Which picture of herself did no girl submit?
b. Her self-portrait.

Engdhal's functional denotation:

(2) $\{\,\textbf{that no girl}_{@}\ x\ \textbf{submitted}\ f(x) \mid \textbf{for all}\ x\ \textbf{,}\ f(x)\ \textbf{is a picture-of}_{@}\ x\,\}$

> THE PROBLEM: as soon as there is any *individual* x, s.t., there is no (actual) picture of x, the question denotation will be empty.

Question LF:

(3) λp [which $\mathbf{E}_1$ picture of herself$_1$] λf ?(p) no girl$_2$ submitted $f(t_2)$

Once we acknowledge the (potential) partiality of the functional variable f, the theory of presupposition projection provided by Heim & Kratzer (1998) (in terms of projecting definedness conditions) delivers a (potentially) partial function as the denotation of the scope-site of *no girls*.

(4) $[\![\lambda x \text{ submitted } f(x)]\!]^w = \lambda x : x \in \mathbf{dom}(f) \,.\, x \textbf{ submitted}_w\ f(x)$ `et`

Heim assumes universal projection from under negative indefinites; the result, when we compose the rest of the question, is a set of potentially partial propositions with universal definedness conditions:[1]

[1] One thing we should bear in mind is that presupposition projection is not necessarily universal from under every quantifier (Fox 2013). We should double-check that the account of functional readings doesn't make odd predictions for quantifiers which don't display universal projection, as Yimei pointed out last time. The theory of projection in Heim & Kratzer (1998) isn't really suitable for this — we'd need to switch to an explicitly trivalent system in order to check the predictions. I'll leave this as a homework exercise.

(5) $$\left\{ \lambda w' \begin{array}{l} : \textbf{every girl}_@ \textbf{ is in dom}(f) \\ . \textbf{ no girl}_@\ x \textbf{ submitted } f(x) \textbf{ in } w' \end{array} \;\middle|\; \textbf{every } x \textbf{ in dom}(f) \textbf{ is s.t., } f(x) \textbf{ is a picture-of}_@\ x \right\}$$

Heim observes that only two kinds of proposition can ever be in the question extension: if f is s.t., $\{ x \mid x \textbf{ is a girl}_@ \} \subseteq \mathbf{dom}(f)$, the proposition is bivalent; if $\{ x \mid x \textbf{ is a girl}_@ \} \nsubseteq \mathbf{dom}(f)$, then we have the unique proposition whose domain is $\varnothing$.

This means we can rewrite the question extension as follows:

(6) $$\left\{ \lambda w' \,.\, \textbf{no girl}_@\ x \textbf{ submitted } f(x) \textbf{ in } w' \;\middle|\; \begin{array}{l} \textbf{every } x \textbf{ in dom}(f) \textbf{ is s.t., } f(x) \textbf{ is a picture-of}_@\ x \\ \textbf{and every girl}_@ \textbf{ is in dom}(f) \end{array} \right\}$$
$$\cup \{ \lambda w \,.\, \# \}$$

> Heim (2012): the presence of the pathological element makes no difference for how the resulting Hamblin set partitions worlds in the context set (given a particular algorithm for partitioning a context set, in light of potential partiality). Before moving further, let's double-check that we understand why.

In order to derive this result, Heim, in a footnote, suggests the following notion of Q-equivalence, which takes into account partiality:

(7) Q-equivalence (partial ver.):
$$w \sim_Q w' = \forall p \in Q[(w \in \mathbf{dom}(p) \wedge p(w)) = (w' \in \mathbf{dom}(p) \wedge p(w'))]$$

We'd like to understand the answers to the following questions:

- Does this make reasonable predictions for presupposition projection in questions more generally?

- Is this crucial to Heim's account?

2.1 Interlude: partiality, partitioning, and contextual restriction

(8) $[\![\text{Aeryn parked her bicycle}]\!]^{w} = \begin{cases} 1 & \textbf{Aeryn has a bicycle and she parked it in } w \\ 0 & \textbf{Aeryn has a bicycle and she didn't park it in } w \\ \text{undefined} & \text{else} \end{cases}$

Once we allow for partial propositions, we must clarify how our algorithm for partitioning a context set deals with partiality.

As a reminder, here's the equivalence relation we assume based on a set of (bivalent) propositions (Groenendijk & Stokhof 1984):

(9) Q-equivalence:
$w \sim_Q w' = \forall p \in Q[p(w) = p(w')]$

The partition of a context set C induced by Q is defined in terms of $\sim_Q$

(10) $\textbf{PART}_Q(C) := \{ \{ w' \mid w \sim_Q w' \wedge w' \in C \} \mid w \in C \}$

For each world w in C, we compute its cell-mates relative to Q by gathering up all the other worlds w' in C that return the same (bivalent) truth-value for every proposition p in Q

Before we consider our reformulation of Q-equivalence, let's first ask what our intuitions are regarding how presuppositions project out of questions:

(11) a. Who parked their bicycle?
$\rightsquigarrow$ *everyone has a bicycle*
b. Which of these three students submitted her p-set early?
$\rightsquigarrow$ *each of the three students prefers* she/her *pronouns*

If intuitions about accommodation are a reliable diagnosis of semantic presupposition,[2], then we intuit that presuppositions project *universally* out of questions, and indeed this is what is typically reported in the literature.[3]

[2] And they're likely not! (Fox 2013).

[3] See e.g., Schlenker 2008.

Heim's suggested notion of Q-equivalence, accommodating partiality.

(12) Q-equivalence (partial ver.):
$w \sim_Q w' = \forall p \in Q[(w \in \textbf{dom}(p) \wedge p(w)) = (w' \in \textbf{dom}(p) \wedge p(w'))]$

Let's first check that this achieves the intended result for functional readings.

Recall the question extension for a functional question:

(13) a. Which of her pictures did no girl submit?

b. $$\left\{ \lambda w' \,.\, \textbf{no girl}_{@}\ x\ \textbf{submitted}\ f(x)\ \textbf{in}\ w' \;\middle|\; \begin{array}{l} \textbf{every } x \textbf{ in dom}(f) \textbf{ is s.t., } f(x) \textbf{ is a picture-of}_{@}\ x \\ \textbf{and every girl}_{@} \textbf{ is in dom}(f) \end{array} \right\}$$
$$\cup \{ \lambda w \,.\, \# \}$$

According to Heim's Q-equivalence, two worlds $w, w' \in C$ are Q-cell-mates *iff* at each $p \in Q$, either:

- w and w' are defined and true, or
- w and w' are each either false or undefined.

Since the pathological member of the question extension is a constant function to undefined, it will clearly never make any difference to cellmate-hood; any two worlds w and w' always return *undefined* for the pathological element.

> Assuming Dayal's presupposition, and a contextual restriction to natural functions, this question should presuppose there is a picture of each girl, that each girl didn't submit exactly one picture of herself. This seems right.

Now, let's see what predictions this equivalence relation makes for "ordinary" presupposition projection in questions:

(14) $$[\![\text{who (of Aeryn and John) parked their bicycle}]\!]^{w} = \{ \lambda w' : x\ \textbf{has a bicycle in}\ w' \,.\, x\ \textbf{parked}\ x\textbf{'s bicycle in}\ w' \}$$
$$= \left\{ \begin{array}{l} \lambda w' \,.\, \textbf{Aeryn has a bicycle in}\ w' \,.\, \textbf{Aeryn parked her bicycle in}\ w', \\ \lambda w' \,.\, \textbf{John has a bicycle in}\ w' \,.\, \textbf{John parked his bicycle in}\ w', \end{array} \right\}$$

Assuming that updating C with an interrogative ϕ *partitions* C relative to $[\![\phi]\!]^{w}$[4], Heim's Q—equivalence relation doesn't predict universal projection.

Consider the following context set, which expresses contextual ignorance about which of John and Aeryn have bicycles, and if they do whether they parked it.

The formulae stand in for worlds:

[4] Where w is an arbitrary world in C, assuming that the askability conditions have been met.

- a_p : Aeryn has a bicycle and parked it; j_p John has a bicycle and parked it;
- $a_{\neg p}$: Aeryn has a bicycle and didn't park it; $j_{\neg p}$: John has a bicycle and didn't park it;
- $a_\varnothing$: Aeryn has no bicycle; $j_\varnothing$: John has no bicycle.

$$C = \left\{ \begin{array}{l} a_p \wedge j_p, a_p \wedge j_{\neg p}, a_p \wedge j_\varnothing \\ a_{\neg p} \wedge j_p, a_{\neg p} \wedge j_{\neg p}, a_{\neg p} \wedge j_\varnothing \\ a_\varnothing \wedge j_p, a_\varnothing \wedge j_{\neg p}, a_\varnothing \wedge j_\varnothing \end{array} \right\}$$

According to Heim's Q-equivalence, two worlds $w, w' \in C$ are Q-cell-mates *iff* at each $p \in Q$, either:

- w and w' are defined and true, or
- w and w' are each either false or undefined.

This means that the question *who parked their bicycle* is predicted to induce the following partition relative to C:

$$\left\{ \begin{array}{lr} \{ a_p \wedge j_p \}, & \text{both Aeryn and John parked their bicycles} \\ \{ a_p \wedge j_{\neg p}, a_p \wedge j_\varnothing \}, & \text{Aeryn parked her bicyle, and John either doesn't have one or didn't park it} \\ \{ a_{\neg p} \wedge j_p, a_\varnothing \wedge j_p \}, & \text{John parked his bicycle, and Aeryn either doesn't have one or didn't park it} \\ \{ a_{\neg p} \wedge j_{\neg p}, a_{\neg p} \wedge j_\varnothing, a_\varnothing \wedge j_{\neg p}, a_\varnothing \wedge j_\varnothing \} & \text{John and Aeryn either don't have bicycles, or didn't park them} \end{array} \right\}$$

Assuming Dayal's presupposition (and that *who* ranges over atoms), at best the question is predicted to presuppose that exactly one of John or Aeryn have a bicycle. This is a bit of a strange result.

> As far as I can see, this is an equivalent result to locally accommodating the presupposition at the question nucleus, and partitioning based on the standard, bivalent definition of Q-equivalence.

$$\left\{ \begin{array}{l} \textbf{that Aeryn has a bicycle and parked it}, \\ \textbf{that John has a bicycle and parked it} \end{array} \right\}$$

Let's try an alternative way of partitioning with partial propositions, which will give rise to universal projection. The idea will be that update with a question presupposes that each possible answer satisfies the Stalnaker's bridge principle (von Fintel 2008).

(15) ϕ is *askable* at C, iff $\forall w, w' \in C[[\![\phi]\!]^w = [\![\phi]\!]^{w'}]$

If a question ϕ is askable at C, then let $Q := [\![\phi]\!]^w$, for an arbitrary $w \in C$. Update of C with ϕ is a partial function:[5]

[5] In order to treat interrogative update as an arrow from contexts to contexts, we can switch to an inquisitive setting; this isn't important for our purposes.

(16) Interrogative update (def.)

$$C[\phi] = \begin{cases} \textbf{PART}_Q(C) & \forall p \in Q, \forall w \in C[p(w) = 1 \vee p(w) = 0] \\ \text{undefined} & \text{else} \end{cases}$$

> For update of C with a question to be defined, update of C with each possible answer in Q should have a chance of being defined.

In fact, we can rewrite as follows, in light of Stalnaker's bridge:

(17) Interrogative update (def.)

$$C[\phi] = \begin{cases} \textbf{PART}_Q(C) & \forall p \in Q[c[p] \text{ is defined}] \\ \text{undefined} & \text{else} \end{cases}$$

This effectively guarantees that every answer is defined at every world in C, and therefore we can stick to our ordinary bivalent notion of Q-equivalence:

(18) Q-equivalence:
$w \sim_Q w' = \forall p \in Q[p(w) = p(w')]$

A question such as *Who parked their bicycle* is of course predicted to presuppose that everyone has a bicycle, thanks to the bridge principle built into our notion of update.[6]

[6] As far as I can tell, this is similar to the notion of interrogative update arrived at by Theiler (2021), on independent grounds.

> When we combine our notion of interrogative update with Heim's question denotation – paying special attention to the pathological member — the results are at first blush disastrous. Update should be always undefined.

In order to ensure reasonable results, the functions that the *wh*-expression ranges over are contextually restricted to *just* those that are defined for all of the girls. In fact, the question denotation coupled with interrogative update *forces* this contextual restriction.[7]

[7] I'm grateful to Filipe for discussing this point with me!

> To conclude, I think that Heim's proposal is in fact compatible with universal projection out of questions, once we take into account the (independently necessary) mechanism of contextual restriction; we've already seen that this was necessary in order to ensure quantification over just the natural functions.

N.b. there are proposals out there that assume a weaker bridge principle (i.e., just one answer need be defined), and aim to derive universal projection via additional pragmatic principles. See, e.g, Schwarz & Simonenko 2018; see Theiler 2021 for an argument that a universal bridge principle is necessary.

3 *Rethinking the functional reading*

3.1 *Background: the copy theory of movement*

> Crucial assumption: the restrictor of the *wh*-expression may be interpreted *in-situ*.

There are two ways of cashing out this conjecture; Heim adopts the second:[8]

[8] See Fox & Johnson 2016 for a hybrid proposal.

- At Logical Form, *which*-phrases are interpreted *in-situ* as definite descriptions (Rullmann & Beck 1998).
- Movement leaves behind a *copy*, which is converted into a *bound definite description* at LF (Fox 1999).

Independent motivation for the Rullmann & Beck 1998 conjecture: *which*-phrases sub-extracted from intensional contexts can be interpreted *de dicto*.

(19) John believes that there is unicorn.
Which unicorn$_i$ does John think $\boxed{\text{that Mary tried to catch the}_i \text{ unicorn}}$?

Cf. projection behaviour of definite descriptions under attitude verbs (Heim 1992) (modulo proviso inferences):

(20) John believes that there is a unicorn and
John thinks $\boxed{\text{that Mary tried to catch the unicorn}}$.

In order to interpret lower copies, we need two type-shifters: Partee's (1986) THE and IDENT.

IDENT is essentially a concretely-typed variant of ?:

(21) $\textbf{IDENT} := \lambda x \,.\, \lambda y \,.\, y = x$ $\quad\langle \texttt{e}, \texttt{et}\rangle$

THE is a covert definite determiner:

(22) $\textbf{THE} := \lambda k \;:\; \exists! x[k(x)] \,.\, \iota x[k(x)]$ $\quad\langle \texttt{et}, \texttt{e}\rangle$

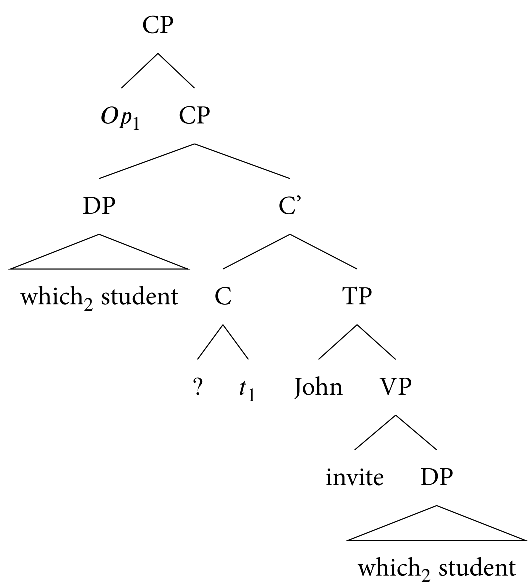
We'll also need to assume that *which* is interpreted as an *unrestricted* existential quantifier:

(23) $[\![\text{which}]\!] = \lambda k \,.\, \exists x[k(x)]$ $\quad\texttt{ett}$

The structure delivered by the narrow syntax for a simple question:

(24) Which student did John invite?

(25)



Schematic algorithm for trace converstion at LF:

(26) a. λp [which$_2$ student] ?(p) John invite [which$_2$ student]
b. λp [which student] λ_2 ?(p) John invite [which 2 student]
$\Rightarrow$ *insert binder and variable*
c. λp [which ~~student~~] λ_2 ?(p) John invite [~~which~~ 2 student]

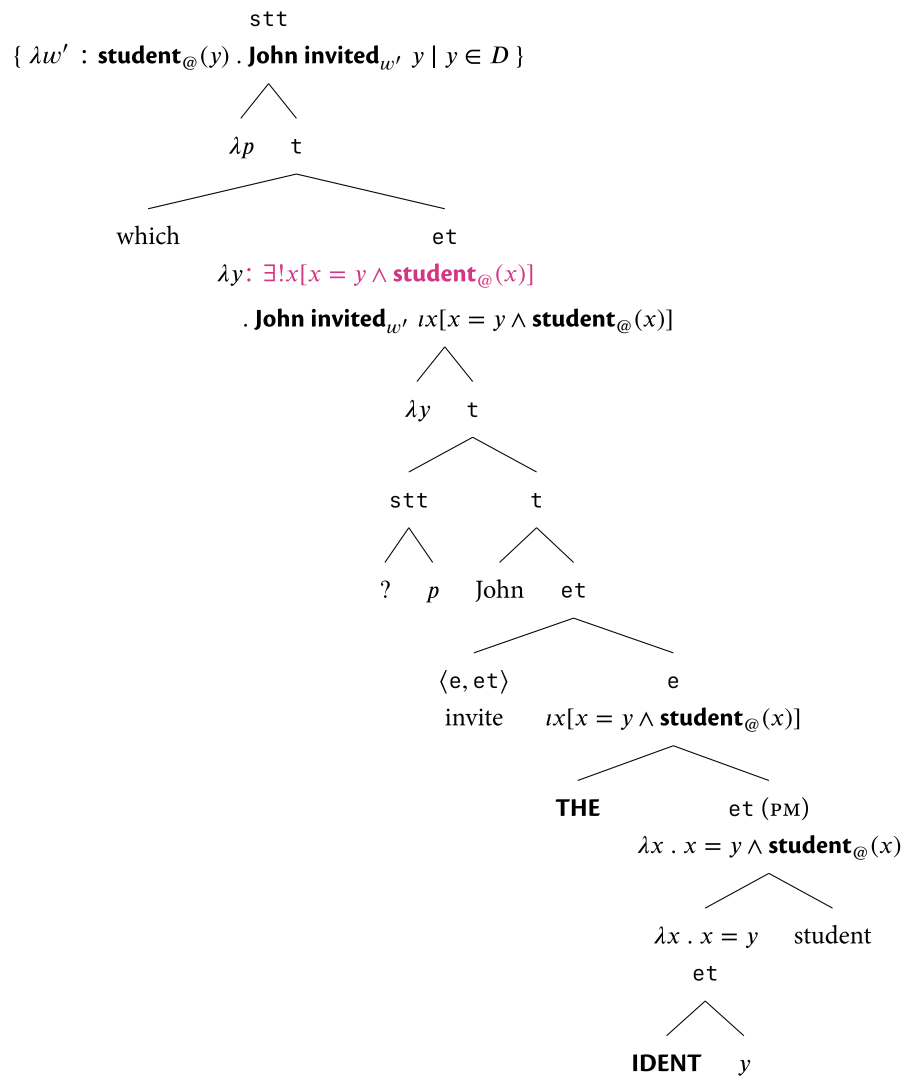
⇒ *delete higher restrictor and lower determiner*

d. λp which λ_2 ?(p) John invite [**THE** [**IDENT** 2] student]

⇒ *Rescue lower copy using type-shifters*

The resulting LF can now be interpreted; LF of the *which*-question post trace conversion:

(27)



> Since the restrictor of the *which*-phrase is interpreted *de re*, the resulting propositions in the question denotation are not really partial; rather, they are either *total* propositions, if y is a student in @, or the unique proposition undefined for any world.

(28) $\{ \lambda w' : \textbf{student}_@(y) \,.\, \textbf{John invited}_{w'}\ y \mid y \in D \}$

This is equivalent to:

(29) $\{\ \lambda w' \,.\ \textbf{John invited}_{w'}\ y \mid \textbf{student}_{@}(y)\ \} \cup \{\ \lambda w \,.\ \#\ \}$

In light of our discussion of partiality and partitioning, this question denotation in fact *forces* the existential introduced by the *wh* to be contextually restricted to just quantification over students, otherwise interrogative update is undefined.

Assuming contextual restriction to just the (actual) students, this naturally collapses to the standard Hamblin denotation of the question.

As acknowledged by Heim, the proposal here is not obviously compatible with the *scope theory of intensionality*; the restrictor in the lower copy is interpreted *de re*, despite occurring within the scope of ?.

3.2 Functional readings via complex copies

(30) Which picture of herself did no girl submit?

THE PLAN: generalize the basic theory to functional readings. We'll need to adopt polymorphic entries for *which*, and the type-shifters responsible for interpreting lower copies, as well as mechanisms for constructing something analogous to layered traces.

Which is a polymorphic existential quantifier, which will allow *which* to quantify over skolem functions.

(31) $[\![\text{which}]\!] \coloneqq \lambda k \,.\ \exists x[k(x)]$ $\quad$ σtt

IDENT takes any value, and returns the (characteristic function of) the singleton set containing that value.

(32) $\textbf{IDENT} \coloneqq \lambda x \,.\ \lambda y \,.\ y = x$ $\quad$ $\langle \sigma, \sigma t \rangle$

THE is a polymorphic definite determiner.

(33) $\textbf{THE} \coloneqq \lambda k \;:\; \exists! x[k(x)] \,.\, \iota x[k(x)]$ $\langle \sigma t, \sigma \rangle$

We'll also need to allow for insertion of covert pronouns, in order to derive something corresponding to a *layered trace*.

The structure of the question (under the functional reading) delivered by the narrow syntax:

(34) λp [which picture of herself$_y$]$_2$
?(p) no girl λy y submit [which$_2$ picture of herself$_y$]

Post TC:

(35) λp which λf
?(p) no girl λy y submit [**THE** [**IDENT** f] picture of herself$_y$]

Rescue via insertion of covert pronoun:

(36) λp which λf
?(p) no girl λy y submit [**THE** [**IDENT** f(**pro**$_y$)] picture of herself$_y$]

Note immediately that the reflexive is *semantically bound* by *no girl*; the reflexive in the higher copy is simply deleted, along with the rest of the restrictor.

A PREDICTION(?): : functional readings of questions should *always* feed condition C violations.

(37) Which picture of himself would John show no girl?
a. The one she wanted to see the most.

(38) Which picture of John would he show no girl?
a. ? The one she wanted to see the most.[9]

[9] The question mark here doesn't indicate my own acceptability judgement, but rather indicates my own uncertainty of the status of this sentence.

Since, post TC:

(39) λp which λf
?(p) no girl λy he show [**THE** [**IDENT** f(**pro**$_y$)] picture of John]

The structure of the lower copy, post trace conversion + insertion of covert pronouns; **pro**$_y$ will eventually be semantically bound by the quantificational subject.

(40)

e

$$\begin{cases} g_f(g_y) & g_y \in \textbf{dom}(g_f) \wedge g_f(g_y)\ \textbf{picture-of}_@\ g_y \\ \text{undefined} & \text{otherwise} \end{cases}$$

[**THE** [$_{et}$ [$_{et}$ **IDENT** [$_{e}$ [$_{ee}$ f] [$_{e}$ **pro**$_y$]]] [$_{et}$ [$_{NP}$ picture of herself$_y$]]]]

As composition proceeds, abstraction over y yields a partial function:

(41) $\lambda y : y \in \textbf{dom}(f) \wedge f(y)\ \textbf{picture-of}_@\ y\ .\ y\ \textbf{submitted}_{w'}\ f(y)$

VP

λy y submitted [**THE IDENT** $f(y)$ picture of y]

The presupposition projects universally through *no girl*:

(42)

TP

$$\begin{cases} \neg\exists y[\textbf{girl}_@(y) \wedge y\ \textbf{submitted}_{w'}\ f(y)] & \forall y[\textbf{girl}_@(y) \rightarrow y \in \textbf{dom}(f) \wedge f(y)\ \textbf{picture-of}_@\ y] \\ \text{undefined} & \text{else} \end{cases}$$

No girl submitted [**THE IDENT** $f(y)$ picture of y]

Again, because the restrictor is interpreted *de re*, the propositions in the answer set are never partially defined.

(43) $\{\ \lambda w'\ .\ \neg\exists y[\textbf{girl}_@(y) \wedge y\ \textbf{submitted}_{w'}\ f(y)] \mid \forall y[\textbf{girl}_@(y) \rightarrow y \in \textbf{dom}(f) \wedge f(y)\ \textbf{picture-of}_@\ y]\ \} \cup$
$\{\ \lambda w'\ .\ \#\ \}$

The result is equivalent to Engdahl. We need to ensure that the *wh*-expression is contextually restricted to just functions defined for every girl, and which return self-pictures.

Homework exercise

Does the same reasoning go through...

- ...if the quantificational NP is interpreted *de dicto*?
- What if the the restrictor of *wh* is interpreted *de dicto*?

3.3 Weak crossover

The theories involve binding into a functional trace, and therefore both predict Weak Crossover (WCO) effects (thanks to Natasha for raising this last time).

(44) a. Which relative of his cares for no Italian male?
b. #His mother.

(45) a. Which friend of his does no Italian male's mother ever care for?
b. His best friend.

Chierchia (1992) argues on the basis of parallel WCO effects with Pair List (PL) readings of questions with universals, that these PL readings are just a special case of functional readings (i.e., giving the graph of the function).

(46) Who does every Italian male love?
a. Paolo, Francesca; Giovanni, Maria.
b. His mother.

(47) who loves every Italian male?
a. #Paolo, Francesca; Giovanni, Maria.
b. #His mother.

Note that once we have the full expressive power of functional readings, it's tempting to try to account for PL readings of multiple questions with functional traces, without recourse to sets of questions.[10]

[10] See Dayal 2017 for discussion.

3.4 Comparison with Engdahl

> One of the main differences between Heim 2012 and Engdahl 1986 is that, on Heim's approach, the reflexive in the restrictor really is (semantically) bound by its antecedent; on Engdahl's approach, the reflexive is *indirectly* bound by **E**.

Evidence for *direct* binding: ϕ-feature transmission (examples from Heim 2012: p. 12):

(48) Which picture of himself/*herself did no boy submit.
(49) Which relative of theirs did most people complain about?
(50) Which mistake that we have made will none of us ever forgive ourselves?

N.b., as Heim acknowledges, the force of this argument depends on the assumption that ϕ-features on bound pronouns/reflexives are determined configurationally (*feature transmission*; Kratzer 2009).

Condition A/B:

(51) Which picture of *himself/him did every boy's mother choose?

4 Addendum: Functional readings without covert pronouns

There's a line of work in Variable Free Semantics (vfs) generalizing a mechanism independently necessary to account for *paycheck pronouns* to functional readings of questions. See, especially Polly Jacobson's work (Jacobson 1999, 2000, 2014).

As shown by Charlow (2019a,b), Jacobson's innovations aren't proprietary to vfs. In the following, I'll attempt to reconstruct Jacobson's analysis of functional readings in a more standard, variable-full setting, based on techniques developed in Charlow 2019a.[11]

[11] This section benefited from discussion with Filipe Hisao-Kobayashi, who independently worked out something similar.

The goal: dispense with covert pronouns.

4.1 Paycheck pronouns

(52) Every philosopher spent his paycheck. Every linguist saved it.

Here, the pronoun it denotes a function that maps individuals to their paychecks.

> How do we account for this compositionally, are pronouns ambiguous? It turns out that we can capture paycheck pronouns by generalizing standard machinery for interpreting variables.

Instead of relativizing the interpretation function to an assignment parameter, we can equivalently enrich our denotations with outer assignment-arguments:

Old system (Heim & Kratzer 1998):

(53) $[\![\text{he}_1]\!]^g = g_1$ e

New system:

(54) $[\![\text{he}_1]\!] = \lambda g \,.\, g_1$ ge

We can define some compositional glue for threading assignments through composition.

Assignment sensitive Function Application (FA) replicates Heim & Kratzer's FA — it performs FA while keeping track of assignment-sensitivity.

(55) Assignment-sensitive FA:
$m \circledast n \coloneqq \lambda g \,.\, m(g)(n(g))$

Pure lifts any value into a trivially assignment sensitive value.

(56) Pure:
$\rho \coloneqq \lambda x \,.\, \lambda g \,.\, x$ $\langle \sigma, g\sigma \rangle$

> This is a correlate of the fact that our old interpretation function was relativized to an assignment parameter, even in the absence of assignment-sensitivity.

We use assignment-sensitive FA and *pure* to compose pronouns with non-assignment-sensitive expressions.

(57) John likes him_1

$\lambda g \,.\, \textbf{john likes}\ g_1$
$\circledast$

- $\lambda g \,.\, \textbf{john}$
  ρ John
- $\lambda g \,.\, \lambda y \,.\, y\ \textbf{likes}\ g_1$
  $\circledast$
  - $\lambda g \,.\, \textbf{likes}$
    ρ likes
  - $\lambda g \,.\, g_1$

We no longer need a syncategorematic rule of *predicate abstraction* — an abstraction operator can be defined categorematically:

(58) Abstraction operator:
$\Lambda_n \coloneqq \lambda k \,.\, \lambda g \,.\, \lambda x \,.\, k(g^{[n\to x]})$ $\langle \mathsf{g}\sigma, \langle \mathsf{g}, \mathsf{e}\sigma\rangle\rangle$

Binding involves insertion of an abstraction operator (Büring 2005):

(59) John Λ_1 spent his$_1$ paycheck.

(60) $\lambda g \,.\, \textbf{John spent}\ (\textbf{paycheck-of John})$

$\circledast$

- $\lambda g \,.\, \textbf{John}$
  - ρ John
- $\lambda g \,.\, \lambda x \,.\, g_1^{[1\to x]}\ \textbf{spent}\ (\textbf{paycheck-of}\ g_1^{[1\to x]})$
  - Λ_1
  - $\lambda g \,.\, g_1\ \textbf{spent}\ (\textbf{paycheck-of}\ g_1)$
    - t_1 spent his$_1$ paycheck

N.b., that in this framework, the denotation of *his paycheck* is an assignment-sensitive individual:

(61) $[\![\text{his}_1\ \text{paycheck}]\!] = \lambda g \,.\, g_1\textbf{'s paycheck}$

So far, we've done nothing except for reconstruct the standard treatment of variables in model-theoretic terms.

> In order to account for paycheck pronouns, we give pronouns a *recursive* type-signature

$$\langle \mathsf{g}_1, \langle \ldots \langle \mathsf{g}_n, \mathsf{e}\rangle\rangle\rangle$$

The meaning of a pronoun doesn't change — it's something that returns a value based on an outer-layer of assignment sensitivity, it's just that sometimes the return value is itself assignment-sensitive.

(62) $[\![\text{pro}_n]\!] \coloneqq \lambda g \,.\, g_n$ $\langle \mathsf{g}_1, \langle \ldots \langle \mathsf{g}_n, \mathsf{e}\rangle\rangle\rangle$

A simple paycheck pronoun is a rigidly typed instantiation of this meaning; given an outer layer of assignment-sensitivity, it returns an assignment-sensitive value:

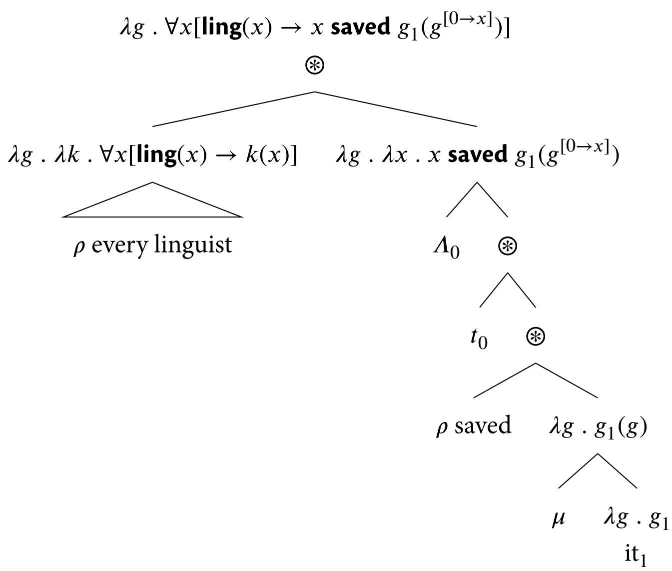
(63) Paycheck pronoun: $[\![\text{it}_1]\!] \coloneqq \lambda g \,.\, g_1$ $\langle \mathsf{g}, \mathsf{ge}\rangle$

In order to incorporate paycheck pronouns into semantic composition, we need one more type-shifter: a flattener (called *join*):

(64) $\mu \coloneqq \lambda i \,.\, \lambda g \,.\, i(g)(g)$ $\langle\langle \mathsf{g}, \mathsf{g}\sigma\rangle, \mathsf{g}\sigma\rangle$

Paycheck pronoun derivation:

(65)



If $g_1 = \lambda g \,.\textbf{paycheck-of}\ g_0$ (the value of "his_0 paycheck" on this theory), we'll get the paycheck reading.

> Let's assume that traces, like pronouns, denote variables, and like pronouns, have a recursive type-signature.

This *predicts* the possibility of a functional reading, just in case the trace of the moved *wh*-expression has a paycheck denotation.

(66) λp Which picture of herself_0 Λ_1 $?(p)$ did no girl Λ_0 submit t_1?

Question nucleus:

(67) $\lambda g \,.\, \lambda i \,.\, p = \lambda w \,.\, \neg\exists x[\textbf{girl}_w(x) \wedge x \textbf{ submit}_w\ i(g^{[0\to x]})]$

- Λ_1
- $\lambda g \,.\, p = \lambda w \,.\, \neg\exists x[\textbf{girl}_w(x) \wedge x \textbf{ submit}_w\ g_1(g^{[0\to x]})]$
  - ρ ?(p)
  - $\lambda g \,.\, \neg\exists x[\textbf{girl}_w(x) \wedge x \textbf{ submit}_w\ g_1(g^{[0\to x]})]$
    - ...
      - ρ no girl
    - $\lambda g \,.\, \lambda x \,.\, x \textbf{ submit}_w\ g_1(g^{[0\to x]})$
      - Λ_0
      - $\lambda g \,.\, g_0 \textbf{ submit}_w\ g_1(g)$
        - t_0
        - $\lambda g \,.\, \lambda x \,.\, x \textbf{ submit}_w\ g_1(g)$
          - ρ submitted
          - $\lambda g \,.\, g_1(g)$
            - μ
            - $\lambda g \,.\, g_1$
              - t_1

Now, we want the *wh*-expression to existentially quantify over assignment-sensitive description, i.e., $\lambda g \,.\, g_0$**'s selfie**, $\lambda g \,.\, g_0$**'s self-portrait**, etc.

If we compose the meaning ordinarily, the *wh*-restrictor will denote an assignment-sensitive predicate:

(68) $[\![\text{picture of herself}_0]\!]^w \coloneqq \lambda g \,.\, \{\, x \mid\ x \textbf{ picture-of } g_0 \,\}$ $\langle \mathtt{g}, \mathtt{et} \rangle$

In order to get the functional reading, we need an operation that will push the assignment-sensitivity inwards. We may as well call it **E**:

(69) $\textbf{E}(P) \coloneqq \{\, i \mid \forall g[i(g) \in P(g)] \,\}$

(70) $[\![\textbf{E} \text{ picture of herself}_0]\!]^w \ \coloneqq\ \{\, i \mid\ \forall g[i(g) \in \{\, x \mid x\textbf{picture-of}_w\ g_0 \,\}] \,\}$
$\langle \mathtt{g}, \mathtt{et} \rangle$

As usual, we can assume that *which* is a polymorphic existential determiner.

The resulting question denotation:

(71) $\{\ \lambda g w'\ .\ w'\ .\ \neg\exists x[\textbf{girl}_{w'}(x) \wedge x\ \textbf{submit}_{w'}\ i(g^{[0\to x]})] \mid \forall g[i(g) \in \{\ x \mid x\ \textbf{picture-of}_{w}\ g_0\ \}]\ \}$

References

Büring, Daniel. 2005. *Binding theory* (Cambridge Textbooks in Linguistics). Cambridge: Cambridge University Press.

Charlow, Simon. 2019a. A modular theory of pronouns and binding. Unpublished manuscript. Rutgers University. https://ling.auf.net/lingbuzz/003720.

Charlow, Simon. 2019b. Variable-free semantics and flexible grammars for anaphora. lingbuzz/004503.

Chierchia, Gennaro. 1992. Functional wh and weak crossover. In *Proceedings of WCCFL 10*, 75–90.

Dayal, Veneeta. 2017. *Questions* (Oxford Surveys in Semantics and Pragmatics). Oxford, New York: Oxford University Press. 352 pp.

Engdahl, E. 1986. *Constituent Questions: The Syntax and Semantics of Questions with Special Reference to Swedish* (Studies in Linguistics and Philosophy). Springer Netherlands. https://www.springer.com/gp/book/9789027719553 (30 March, 2021).

Fox, Danny. 1999. Reconstruction, Binding Theory, and the Interpretation of Chains. *Linguistic Inquiry* 30(2). 157–196.

Fox, Danny. 2013. Presupposition projection from quantificational sentences - Trivalence, local accommodation, and presupposition strengthening. In Ivano Caponigro & Carlo Cecchetto (eds.), *From grammar to meaning*, 201–232.

Fox, Danny & Kyle Johnson. 2016. QR is restrictor sharing. In Kyeong-min Kim et al. (eds.), *Proceedings of the 33rd West Coast Conference on Formal Linguistics*, 1–16. Somerville, MA: Cascadilla Proceedings Project. http://www.lingref.com/cpp/wccfl/33/.

Groenendijk, Jeroen a. G. & Martin J. B. Stokhof. 1984. *Studies on the semantics of questions and the pragmatics of answers*. Institute for Logic, Language and Computation (ILLC) dissertation. http://dare.uva.nl/record/1/392528.

Heim, Irene. 1992. Presupposition projection and the semantics of attitude verbs. *Journal of Semantics* 9(3). 183–221. https://academic.oup.com/jos/article-lookup/doi/10.1093/jos/9.3.183.

Heim, Irene. 2012. Functional readings without type-shifted noun phrases. Unpublished manuscript. MIT. https://semanticsarchive.net/Archive/TUwYzUwZ/functional%20readings.pdf.

Heim, Irene & Angelika Kratzer. 1998. *Semantics in generative grammar* (Blackwell Textbooks in Linguistics 13). Malden, MA: Blackwell. 324 pp.

Jacobson, Pauline. 1999. Towards a Variable-Free Semantics. *Linguistics and Philosophy* 22(2). 117–184.

Jacobson, Pauline. 2000. Paycheck Pronouns, Bach-Peters Sentences, and Variable-Free Semantics. *Natural Language Semantics* 8(2). 77–155. https://doi.org/10.1023/A:1026517717879 (1 April, 2021).

Jacobson, Pauline. 2014. *Compositional semantics: an introduction to the syntax/semantics interface* (Oxford Textbooks in Linguistics). New York, NY: Oxford University Press. 427 pp.

Kratzer, Angelika. 2009. Making a pronoun: Fake indexicals as windows into the properties of pronouns. *Linguistic Inquiry* 40(2). 187–237.

Partee, Barbara. 1986. Noun-phrase interpretation and type-shifting principles. In J. Groenendijk, D. de Jongh & M. Stokhof (eds.), *Studies in discourse representation theory and the theory of generalized quantifiers*, 115–143. Dordrecht: Foris.

Rullmann, Hotze & Sigrid Beck. 1998. Presupposition Projection and the Interpretation of which -Questions. *Semantics and Linguistic Theory* 8(0). 215–232. https://journals.linguisticsociety.org/proceedings/index.php/SALT/article/view/2811 (11 March, 2021).

Schlenker, Philippe. 2008. Be Articulate: A pragmatic theory of presupposition projection. *Theoretical Linguistics* 34(3). https://www.degruyter.com/view/j/thli.2008.34.issue-3/thli.2008.013/thli.2008.013.xml (13 September, 2020).

Schwarz, Bernhard & Alexandra Simonenko. 2018. Decomposing universal projection in questions. *Proceedings of Sinn und Bedeutung* 22(2). 361–374. https://ojs.ub.uni-konstanz.de/sub/index.php/sub/article/view/111 (8 April, 2021).

Theiler, Nadine. 2021. An Epistemic Bridge for Presupposition Projection in Questions. *Semantics and Linguistic Theory* 30(0). 252–272. https://journals.linguisticsociety.org/proceedings/index.php/SALT/article/view/30.252 (8 April, 2021).

von Fintel, Kai. 2008. What Is Presupposition Accommodation, Again?*. *Philosophical Perspectives* 22(1). 137–170. https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1520-8583.2008.00144.x (18 November, 2020).